St Teresa's RC Primary School
Concrete / Pictorial / Abstract
Maths Calculation Policy

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils, it is important that that the mathematical language used in maths lessons reflects the vocabulary used throughout this policy.

Recommended practice delivering a mastery approach

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support children with their examinations and throughout secondarys.

We are using the White Rose Hub philosophy of:

- fluency - using Learning Objectives from the National Curriculum
- reasoning
- problem-solving

In all our maths work we are using a CPA approach within our maths lessons (CPA – Concrete/ Pictorial/ Abstract)

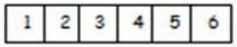
We are using resources such as - White Rose, NCETM Mastery documents & nrich problems.

The aim is that when children leave St Teresa's RC Primary School they:

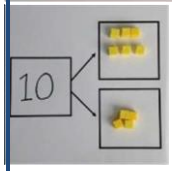
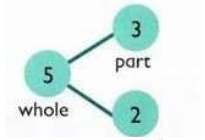
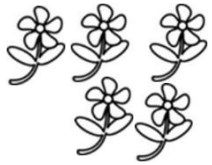
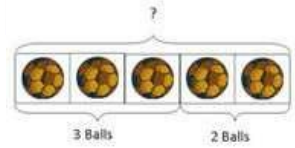
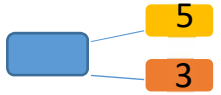
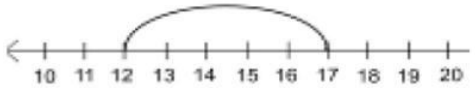
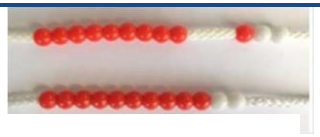
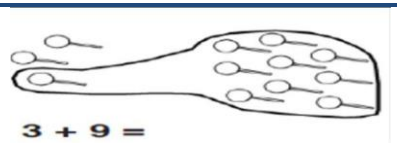
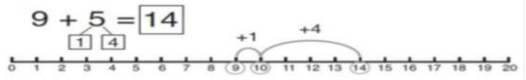
- Have a secure knowledge of number facts and a good understanding of the four calculation operations (addition, subtraction, multiplication and division)
- Make use of jottings, diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads
- Have an efficient, reliable, written method of calculation for each operation that they are able to apply with confidence when they are unable to perform a calculation mentally

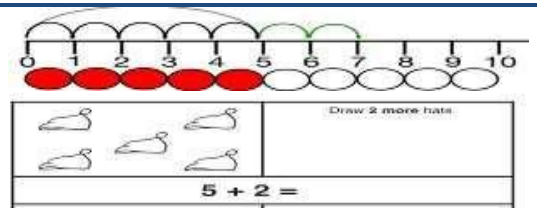
Addition – EYFS

Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

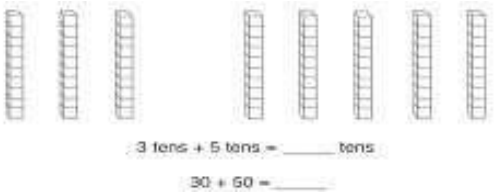
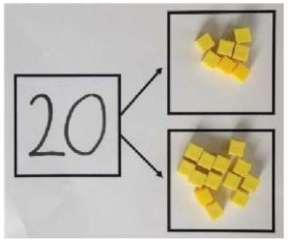
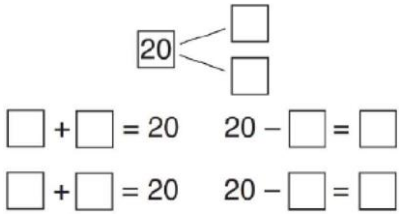
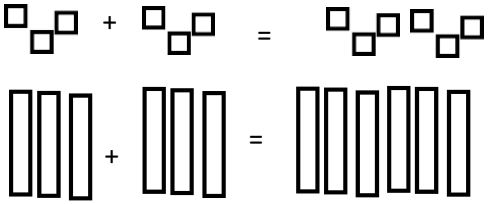
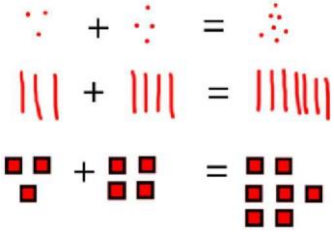
GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
If available, Numicon shapes are introduced straight away and can be used to: • identify 1 more/less • combine pieces to add. • find number bonds. • add without counting.  Children can record this by printing or drawing around Numicon pieces. Children begin to combine groups of objects using concrete apparatus  Construct number sentences verbally or using cards to go with practical activities. Children are encouraged to read number sentences aloud in different ways "Three add two equals 5" "5 is equal to three and two" Children make a record in pictures, words or symbols of addition activities already carried out. Solve simple problems using fingers  5 + 1 = 6 Number tracks can be introduced to count up on and to find one more:  What is 1 more than 4? 1 more than 13? Number lines can then be used alongside number tracks and practical apparatus to solve addition calculations and word problems.  **Children will need opportunities to look at and talk about different models and images as they move between representations.**	**Games and songs can be a useful way to begin using vocabulary involved in addition e.g.** Alice the Camel add more and make sum total altogether score double one more, two more, ten more... how many more to make...? how many more is... than...?

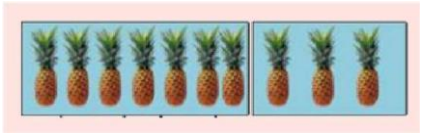
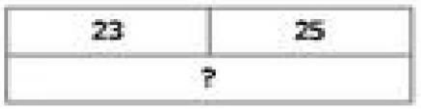
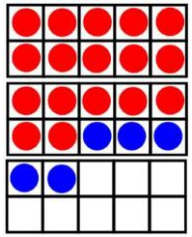
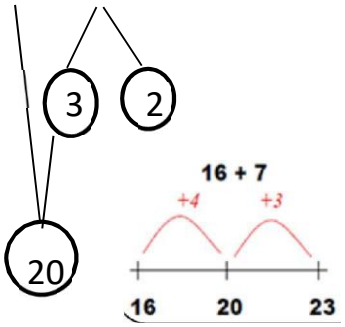
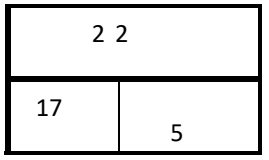
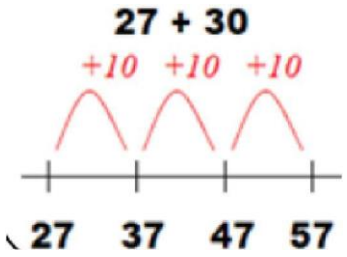
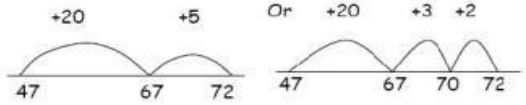
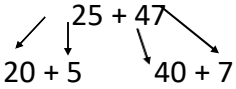
YEAR 1 Addition

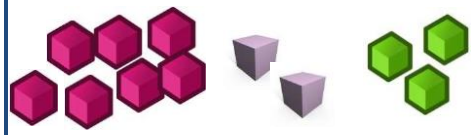
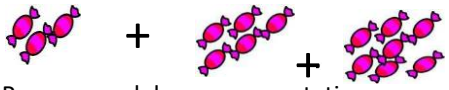
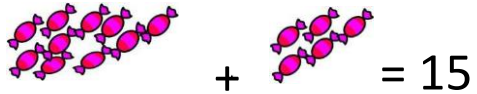
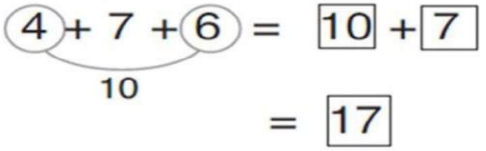
Objective / Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	  Use part, part whole model. Use cubes to add two numbers together as a group or in a bar.  	Use pictures to add two numbers together as a group or in a bar.    	8 = 5 + 3 5 + 3 = 8  Use the part part whole diagram as shown above to move into the abstract. Include missing number questions to support varied fluency: 8 = ? + 3 5 + ? = 8
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	 12 + 5 = 17 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames.	 Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10? How many more do I add on now?

Represent & use number bonds and related subtraction facts within 20	 2 more than 5.		Include missing number questions: 8 = ? + 3 5 + ? = 8 Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

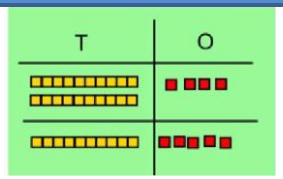
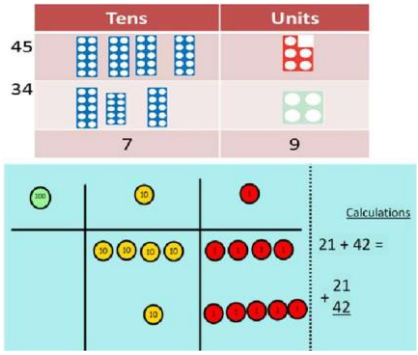
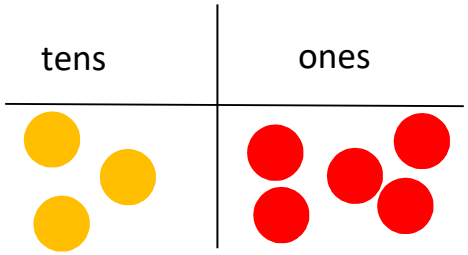
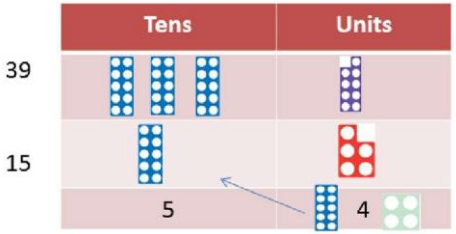
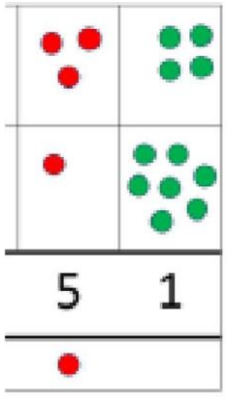
YEAR 2 Addition

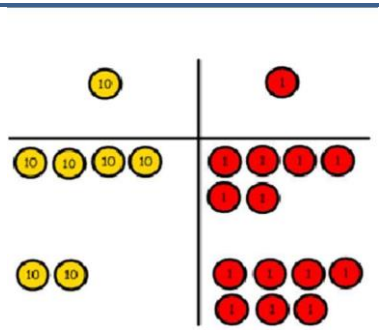
Objective /Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part, part whole	 Children explore ways of making numbers within 20		Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations. 
Using known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700

Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make ‘magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	17 + 5 = 22 Use part part whole and number line to model.  	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17  Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change	 	27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	 20 + 40 = 60 5+ 7 =12 60 + 12 = 72

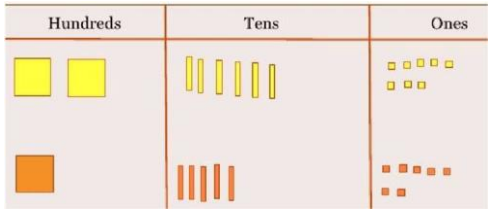
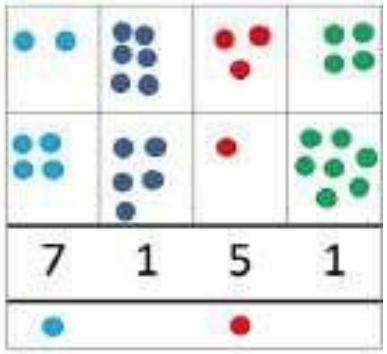
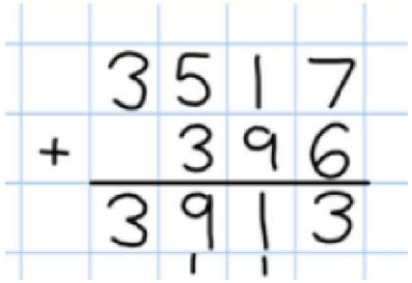
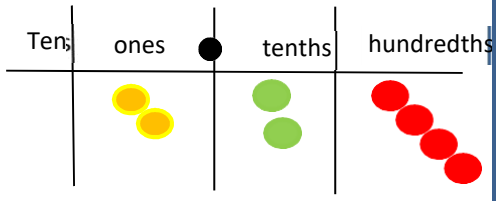
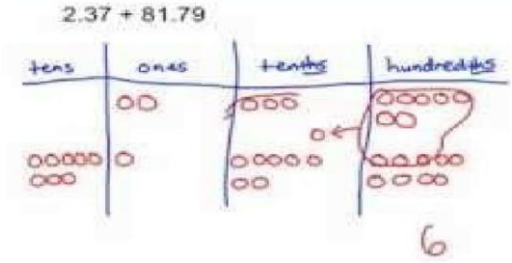
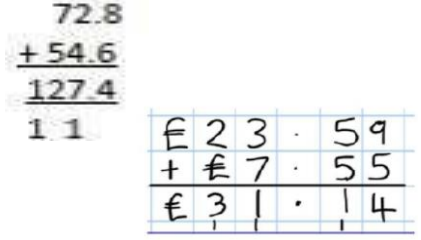
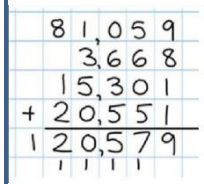
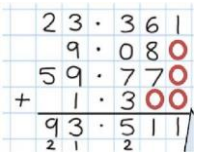
			Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values.
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation. 	 Combine the two numbers that make/ bridge ten then add on the third.

YEAR 3 Addition

Objective /Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3digit numbers.	 Dienes or numicon Add together the ones first, then the tens.  Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	$\begin{array}{r} 223 \\ +\ 114 \\ \hline 337 \end{array}$ Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and place value counters.	 $\begin{array}{r} 34 \\ +17 \\ \hline \end{array}$ Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	$\begin{array}{rcl} 20 & + & 5 \\ 40 & + & 8 \\ \hline 60 & + & 13 \end{array} = 73$ Start by partitioning the numbers before formal column to show the exchange. $\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ \hline 11 \end{array}$

	 46 + 27 = 73		
Estimate the answers to questions and use inverse operations to check answers	 Estimating 98 + 17 = ? 100 + 20 = 120	Use number lines to illustrate estimation. 	Building up known facts and using them to illustrate the inverse and to check answers: 98 + 18 = 116 116 – 18 = 98 18 + 98 = 116 116 – 98 = 18

YEARS 4 – 6 Addition

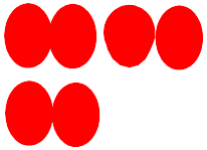
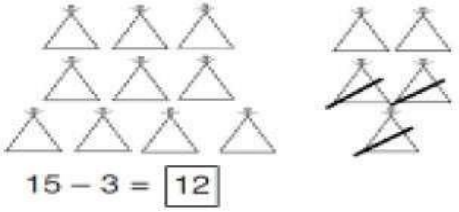
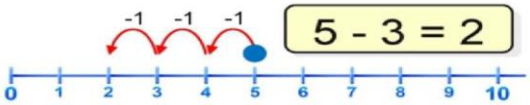
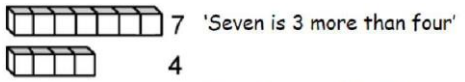
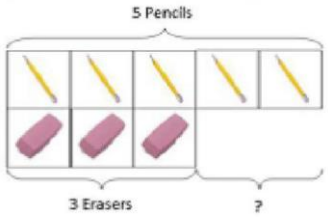
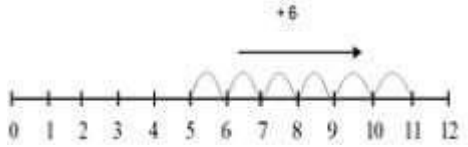
Objective /Strategy	Concrete	Pictorial	Abstract
Years 4 – 6 Estimate and use inverse operations to check answers to a calculation	**AS per Year 3**		
Y4—add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using place value grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.		
Y6—add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	Insert zeros for place holders.  

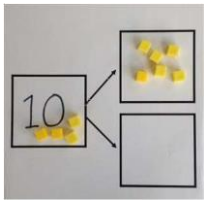
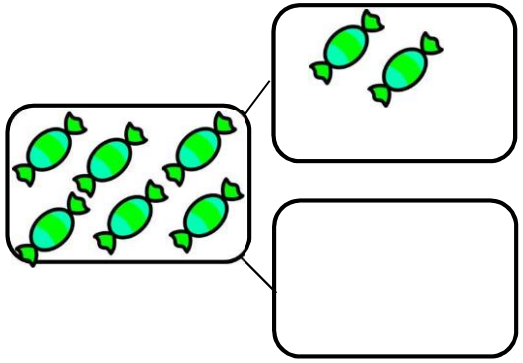
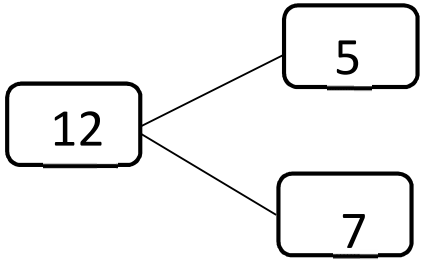
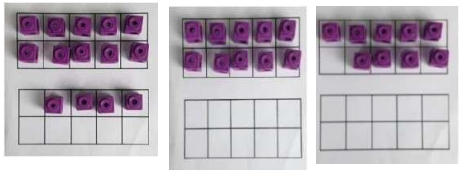
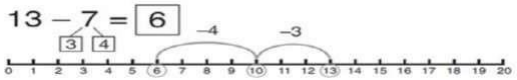
Subtraction - EYFS

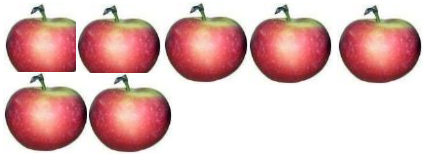
Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
Children begin with mostly pictorial representations XXX XX Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left. Concrete apparatus models the subtraction of 2 objects from a set of 5. Construct number sentences verbally or using cards to go with practical activities. Children are encouraged to read number sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one" Children make a record in pictures, words or symbols of subtraction activities already carried out. Solve simple problems using fingers Number tracks can be introduced to count back and to find one less: 1 2 3 4 5 6 What is 1 less than 9? 1 less than 20? Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back under the number line. **Children will need opportunities to look at and talk about different models and images as they move between representations.**	**Games and songs can be a useful way to begin using vocabulary involved in subtraction e.g.** Five little men in a flying saucer take (away) leave how many are left/left over? how many have gone? one less, two less... ten less... how many fewer is... than...? difference between is the same as

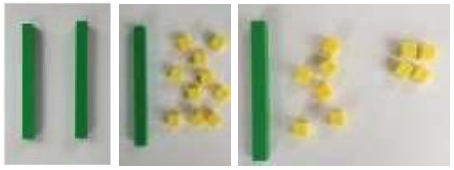
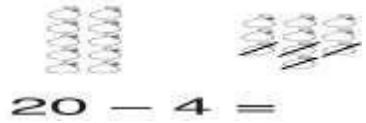
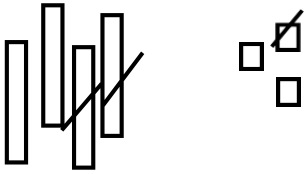
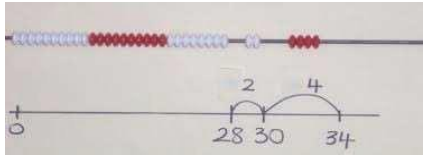
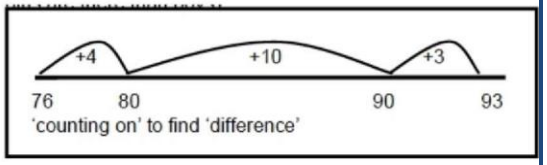
YEAR 1 SUBTRACTION

Objective /Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away. 4—2 = 2   6—4 = 2	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	7—4 = 3 16—9 = 7
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts  'Seven is 3 more than four' 'I am 2 years older than my sister'  Lay objects to represent bar model.	Count on using a number line to find the difference.  	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.?

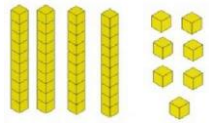
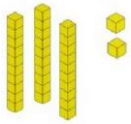
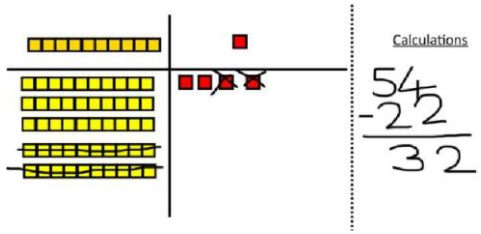
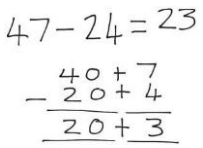
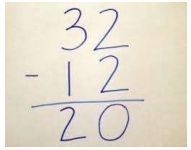
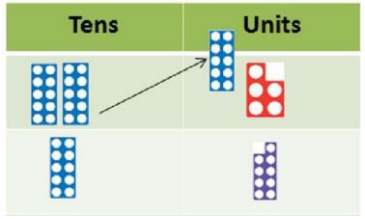
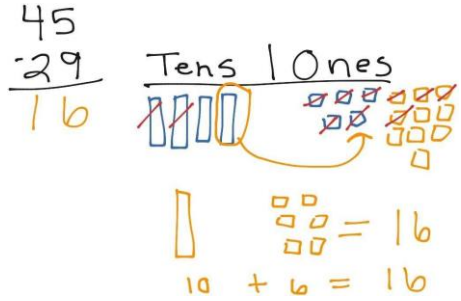
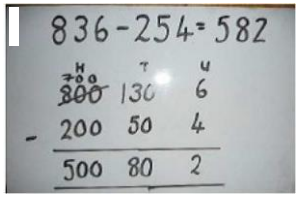
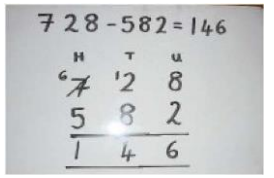
Objective/Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 **Include subtracting zero** Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model.  Include missing number problems: 12 - ? = 5 7 = 12 - ?
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	 13—7 Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?

Bar model Including the inverse operations.	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

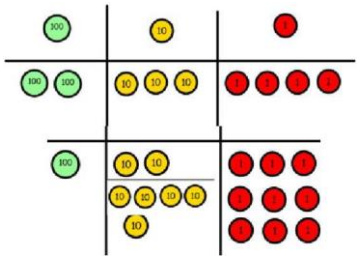
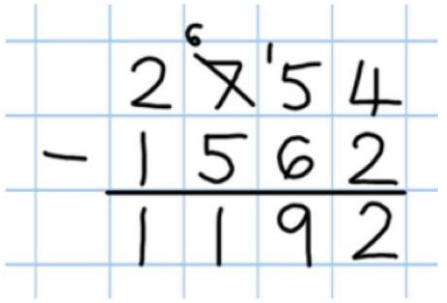
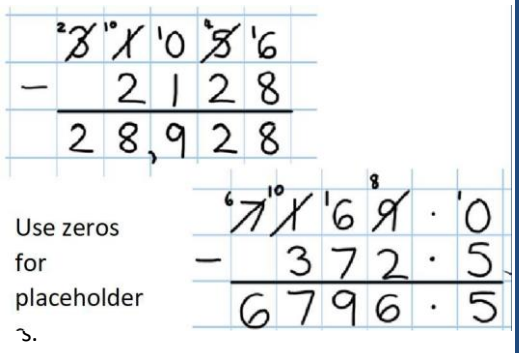
YEAR 2 - SUBTRACTION

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'		20—4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17

YEAR 3 - SUBTRACTION

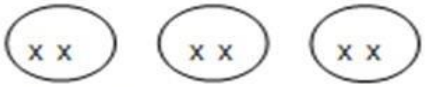
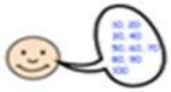
Objective/ Strategy	Concrete	Pictorial	Abstract
Subtract numbers mentally, including: three digit number + ones three digit number + tens three digit number + hundreds			Vary the position of the answer and question. Expose children to missing number questions and vary the missing part of the calculation. 678 = ? - 1 688 – 10 = ? 678 = ? – 100
Column subtraction without regrouping (friendly numbers)	  Use base 10 or Numicon to model	 Draw representations to support understanding	 Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase ‘take and make’ for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method.

YEARS 4 – 6 SUBTRACTION

Objective /Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal Up to 3 decimal places*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for placeholder s.
Year 6—Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).	As Year 4	Children to draw pv counters and show their exchange—see Y3	

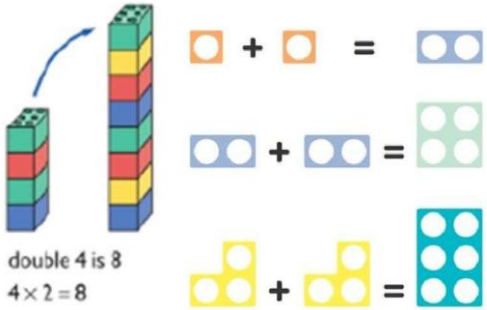
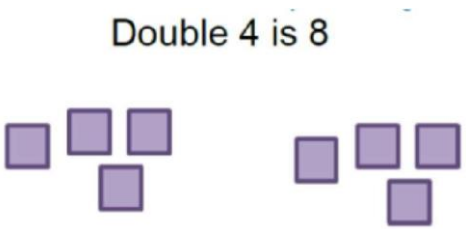
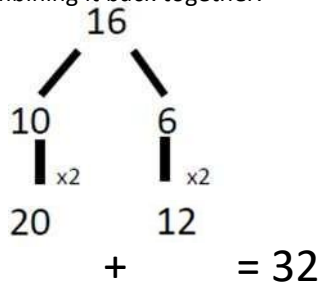
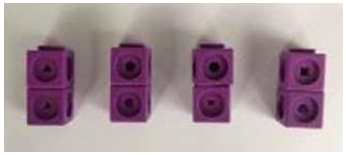
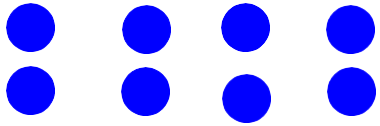
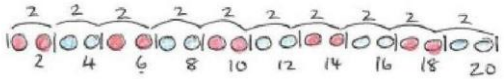
Multiplication - EYFS

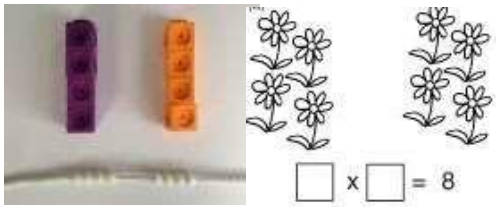
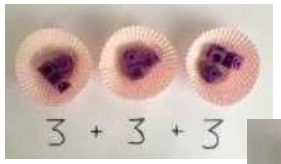
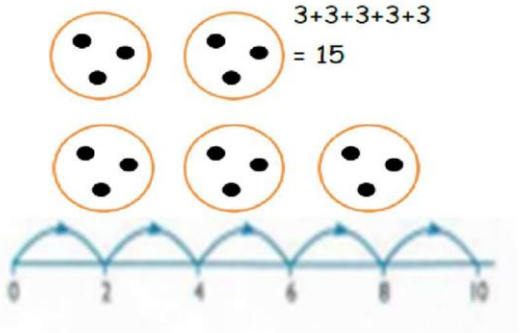
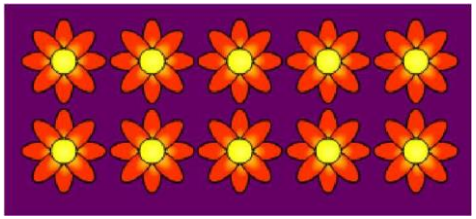
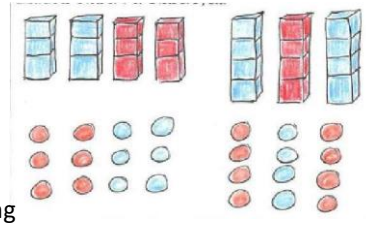
Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
The link between addition and multiplication can be introduced through doubling. If available, Numicon is used to visualise the repeated adding of the same number. These can then be drawn around or printed as a way of recording. Children begin with mostly pictorial representations:   How many groups of 2 are there? Real life contexts and use of practical equipment to <u>count in repeated groups of the same size</u>:  How many wheels are there altogether?  How much money do I have? Count in twos; fives; tens both aloud and with  objects Children are <u>given multiplication problems set in a real life context</u>. Children are encouraged to visualise the problem. How many fingers on two hands? How many sides on three triangles? How many legs on four ducks? Children are encouraged to read number sentences aloud in different ways "five times two makes ten" "ten is equal to five multiplied by two"	lots of groups of times multiply multiplied by multiple of once, twice, three times... ten times... ...times as (big, long, wide... and so on) repeated addition double

YEAR 1 MULTIPLICATION

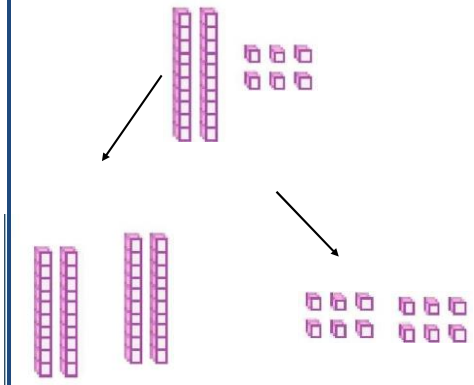
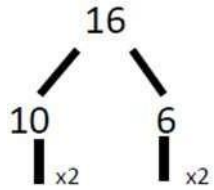
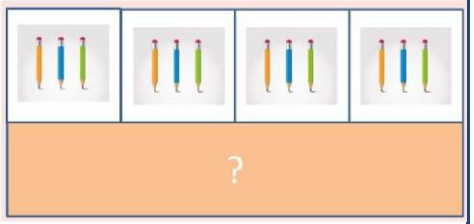
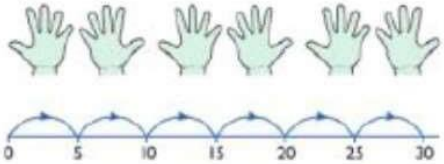
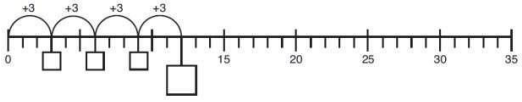
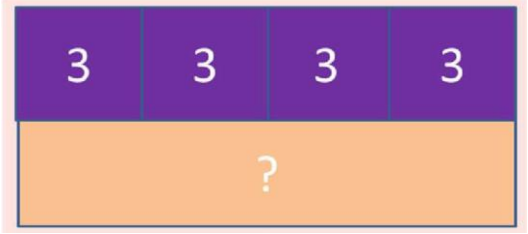
Programme of Study specifies the following objectives, however it does not require the explicit teaching of the mathematical symbol of multiplication

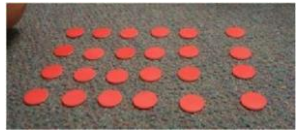
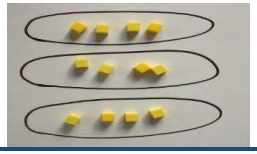
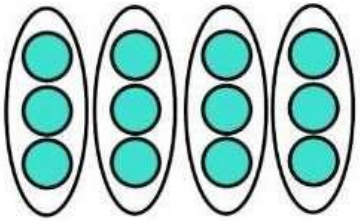
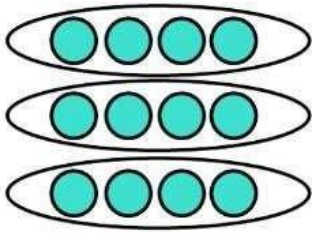
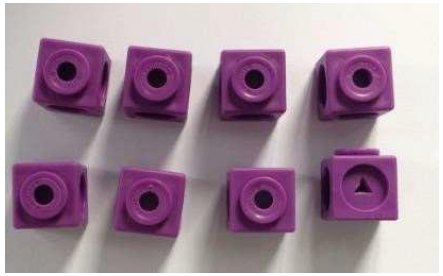
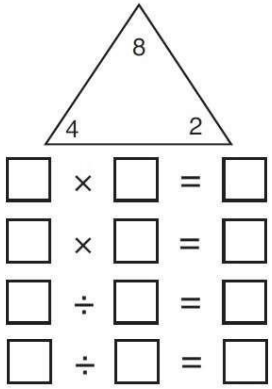
Objective / Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipultives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each part before recombining it back together. 
Counting in multiples (2s, 5s, 10s)	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Making equal groups and counting the total	 Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8
Repeated addition	   Use different objects to add equal groups	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10

YEAR 2 MULTIPLICATION

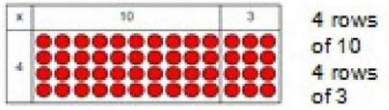
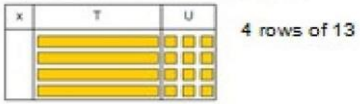
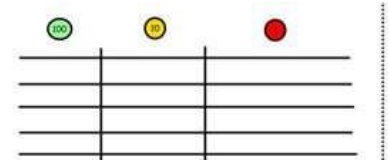
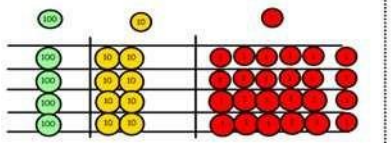
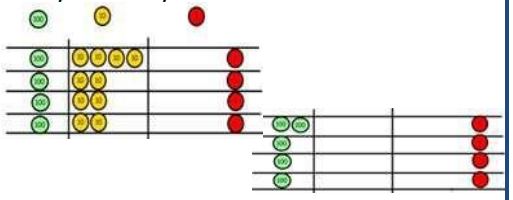
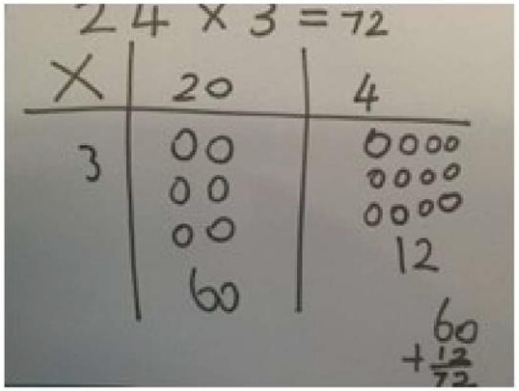
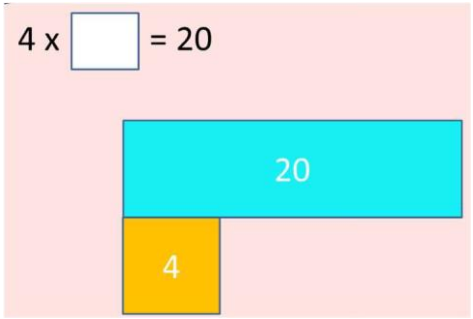
Children should be able to recall and use multiplication and division facts for the 2, 5 and 10 times times tables.

Objective / Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together.  20 + 12 = 32
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.   5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.    	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

Objective / Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and  Numicon.   Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

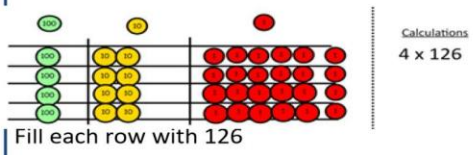
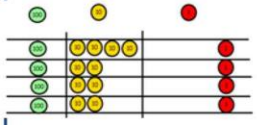
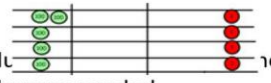
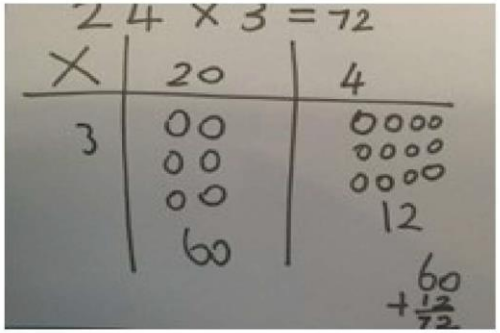
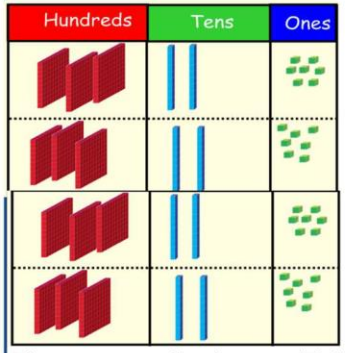
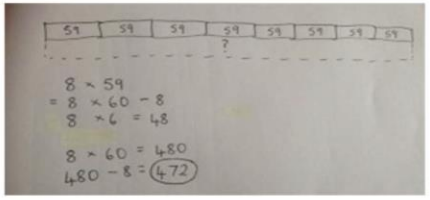
YEAR 3 MULTIPLICATION

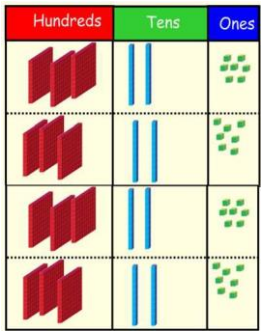
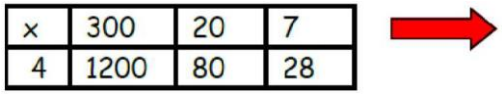
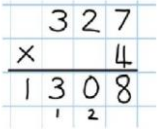
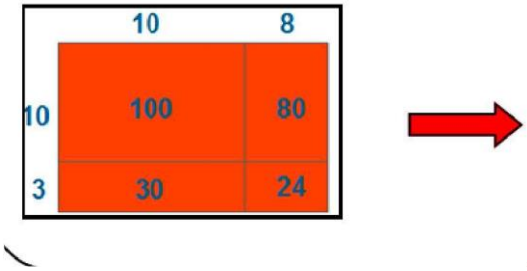
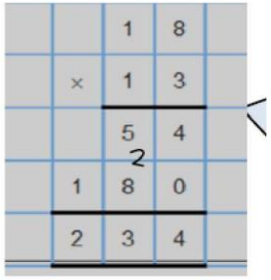
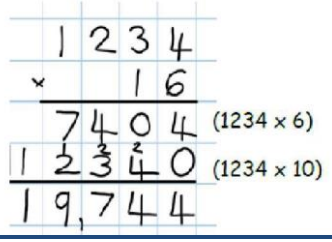
Children should be able to recall and use multiplication facts for the 3,4, and 8 times tables

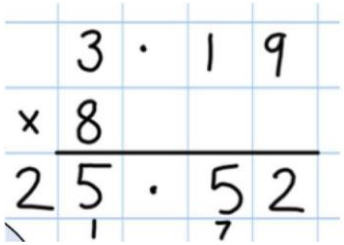
Objective /Strategy	Concrete	Pictorial	Abstract
Grid method, progressing to the formal method Multiply 2 digit numbers by 1 digit numbers	Show the links with arrays to first introduce the grid method.  Move onto base ten to move towards a more compact method.  Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows   Fill each row with 126. Add up each column, starting with the ones making any exchanges needed Then you have your answer. 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  Bar model are used to explore missing numbers  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
---	---	---
7	210	35	 210 + 35 = 245 Move forward to the formal written method: 35 X 7 245 3

Solve problems, including missing number problems, integer scaling problems,			Three times as high, eight times as long ? x 5 = 20 20 ÷ ? = 5 3 hats and 4 coats, how many different outfits?

YEARS 4 – 6 Multiplication

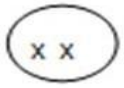
Objective /Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126 Fill each row with 126   Add up each colu…nes making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | 7 | 210 | 35 | 210 + 35 = 245
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642  It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	| × | 300 | 20 | 7 | | 4 | 1200 | 80 | 28 | The grid method my be used to show how this relates to a formal written method.  Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	327 x 4 28 80 1200 1308 3 2 7 × 4 1 3 0 8 1 2 This may lead to a compact method.

Objective /Strategy	Concrete	Pictorial	Abstract
Column Multiplication for 3 and 4 digits x 1 digit.	 It is important at this stage that they always Multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642		327 x 4 28 80 120 1308 
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 Continue to use bar modelling to support problem solving	 18 x 3 on the first row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first 

Objective/Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

Division and fractions - EYFS

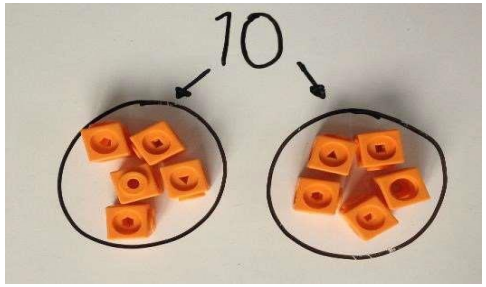
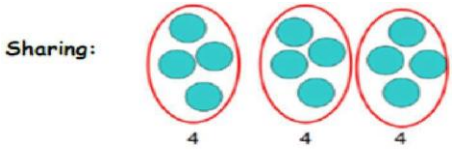
Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

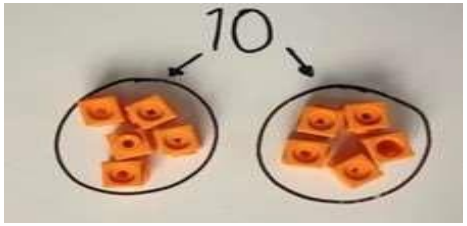
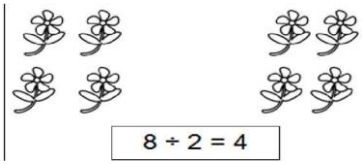
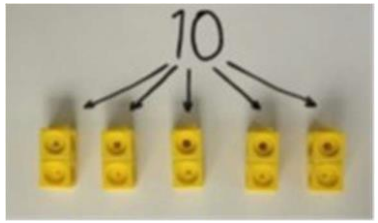
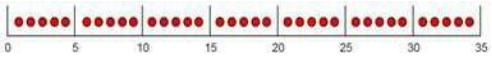
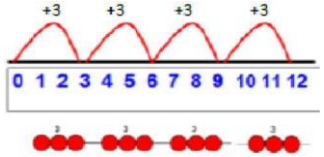
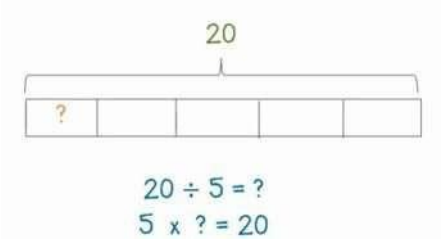
GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
The ELG states that children **solve problems, including doubling, halving and sharing.** Children need to see and hear representations of division as both grouping and sharing. Division can be introduced through halving. Children begin with mostly pictorial representations linked to real life contexts:    **Grouping model** Mum has 6 socks. She grouped them into pairs – how many pairs did she make?  **Sharing model** I have 10 sweets. I want to share them with my friend. How many will we have each? Children have a go at recording the calculation that has been carried out.	halve share, share equally one each, two each, three each… group in pairs, threes… tens equal groups of divide divided by divided into left, left over

FRACTIONS

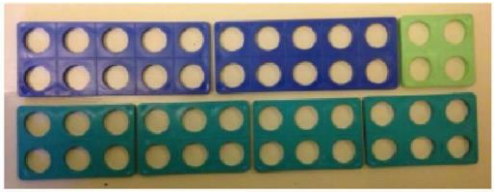
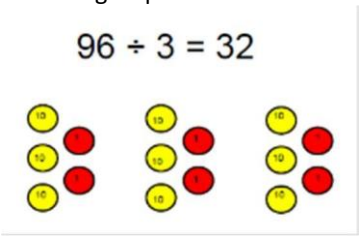
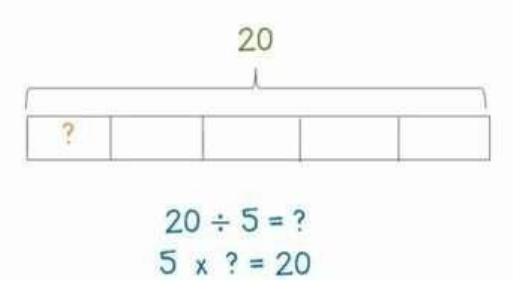
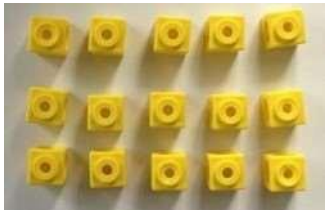
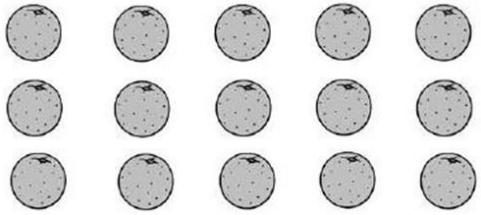
GUIDANCE / MODELS AND IMAGES	KEY VOCABULARY
Although not explicit in the Development Matters document, the sharing model is a useful way of introducing young children to fractions and calculating with fractions. Setting the problems in real life context and solving them with concrete apparatus will support children's understanding. "I have got 5 bones to share between my two dogs. How many bones will they get each?"  Children have a go at recording the calculation that has been carried out. 2 ½ + 2 ½ = 5	As division vocabulary plus: fraction half halves third thirds

YEAR 1			
Objective /Strategy	**Concrete**	**Pictorial**	**Abstract**

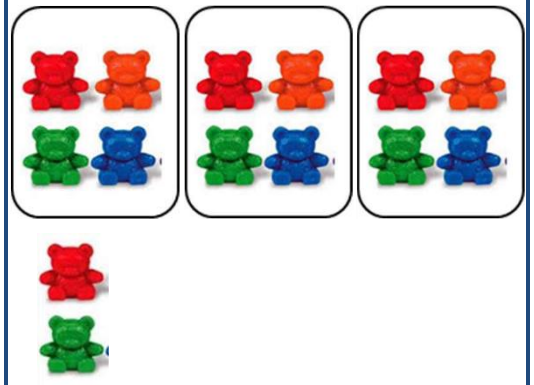
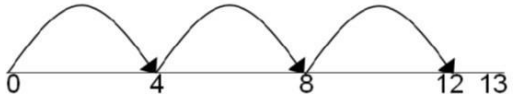
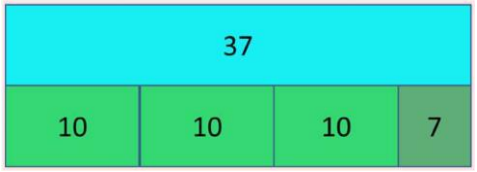
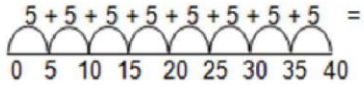
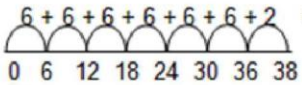
Objective/ Strategy	**Concrete**	**Pictorial**	**Abstract**
Division as sharing *Use Gordon ITPs for modelling*	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is4  12 shared between 3 is 4	12 shared between 3 is 4

Objective/Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 ÷ 3 = 4
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

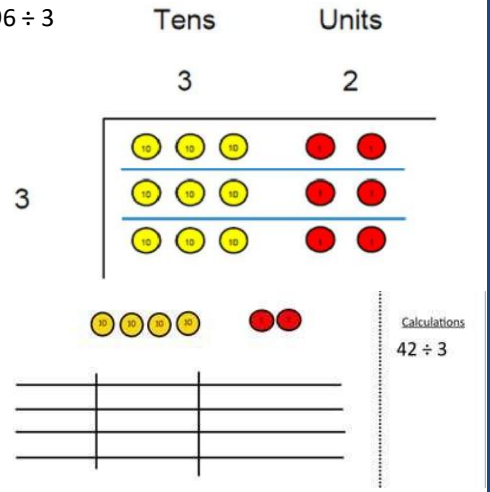
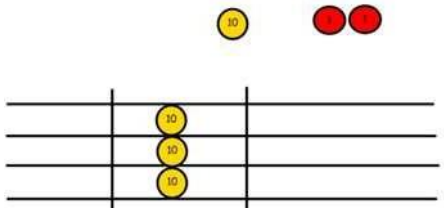
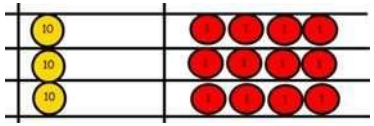
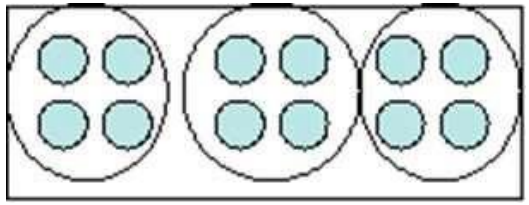
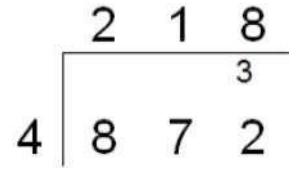
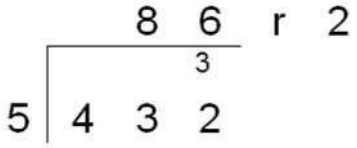
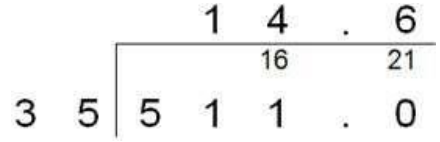
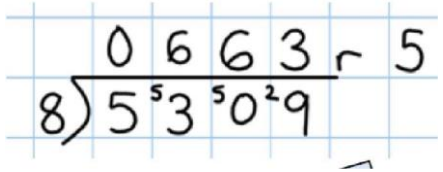
YEAR 2

Objective/Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

YEAR 3 (Greater Depth Y2)

Objective/Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  remainder: 5s in 40?"  = 8 fi mainder:  = 6 sixes with rs, when it becomes inefficient to count in single mu orded using known facts.	Complete written divisions and show the remainder using r. 

Year 4-6

Objective/Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3  Use place value counters to divide using the bus stop method alongside 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  

Long Division

Step 1—a remainder in the ones

```
    h  t  o
      0 4 1 R1
   ---------
4 ) 1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
     th h t  o
       0 4 0 0 R7
   -------------
8 ) 3 2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

Step 1 continued...

```
   h  t  o
      0  6  1
   ------------
4 ) 2  4  7
     - 4
     ------
        3
```

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

```
   th  h  t  o
       0  4  0  2
   ---------------
4 ) 1  6  0  9
      - 8
      ------
         1
```

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1{,}609$

Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 - 4 1	t o 2 9 2)5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 - 4 1 8	t o 2 9 2)5 8 - 4 1 8 - 1 8 0	t o 2 9 2)5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Long Division

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8	h t o 1 2)2 7 8 -2 0	h t o 1 8 2)2 7 8 -2 ↓ 0 7
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 -2 0 7	h t o 1 3 2)2 7 8 -2 0 7 - 6 1	h t o 1 3 2)2 7 8 -2 0 7 - 6 1 8
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0	h t o 1 3 9 2)2 7 8 -2 0 7 - 6 1 8 -1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.

Step 2—a remainder in any of the place values